Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_



**UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

**End Semester Examination – Nov/Dec – 2016**

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|  |  | **Semester :** | **2016-17 ODD** |
| **Code :** | **15MA3012** | **Duration :** | **3hrs** |
| **Sub. Name :** | **FUNCTIONAL ANALYSIS** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| Q. No | Sub Div. | Questions | Course  Outcome | Marks |
| 1. | a. | Let T be a linear operator. Then prove that  a) The range R(T) is a vector space.  b) If , then .  c) The null space N(T) is a vector space. | CO1 | 10 |
| b. | Let X, Y be vector space, both real or both complex. Let be a linear operator with domain  and range . Then prove that  a) The inverse  exists if and only if Tx = 0 x = 0.  b) If  exists, it is a linear operator. | CO1 | 10 |
| (OR) | | | | |
| 2. | a. | If  be a bounded linear operator, where *D(T)* lies in a normed space X and Y is Banach space. Then prove that T has an extension  where is bounded linear operator of norm | CO1 | 15 |
| b. | Prove that finite dimensional vector space is algebraically reflexive. | CO1 | 5 |
| 3. | a. | Prove dual space of is , here  and . | CO2 | 15 |
|  | b. | Let X be normed space and let  be any element of X. Then prove there exists a bounded linear functional  on X such that . | CO2 | 5 |
| (OR) | | | | |
| 4. |  | State and prove Baire’s Category theorem. And hence prove Uniform Boundedness Theorem. | CO2 | 20 |
| 5. |  | State and prove Open Mapping Theorem. | CO2 | 20 |
| (OR) | | | | |
| 6. | a. | State and prove Banach fixed point theorem. | CO2 | 10 |
|  | b. | Let *f(x, y)* be a continuous function of 2 variables in a rectangle and satisfy the Lipschitz condition of order 1 in the second variable y. Further, let be an interior point of A. Then prove that the differential equation has a unique solution say y = g(x) which passes through. | CO2 | 10 |
| 7. |  | A Banach space is a Hilbert Space if and only if its norm satisfies the parallelogram law. | CO3 | 20 |
| (OR) | | | | |
| 8. | a. | State and prove Gram-SchmitOrthogonalization. | CO3 | 10 |
|  | b. | Prove that  is the inner product space but not Hilbert Space for . | CO3 | 10 |
|  | | **Compulsory:** |  |  |
| 9. | a. | State and prove Riesz Representation Theorem. | CO3 | 10 |
|  | b. | Define adjoint operator. Prove that adjoint operator always exists, bounded, linear and unique. | CO3 | 10 |

ALL THE BEST